MATH 202 Differential Equations
Exam 2, Spring 2019
Duration: 60 minutes

Problem	1	2	3	4	5	6	Total
Points	15	15	12	30	15	13	100

Please circle your section:

Lecture 1 MWF 3
Moufawad

Lecture 2 MWF 11
Yamani

Lecture 3 MWF 10
Sabra

Lecture 4 MWF 2
Andrist

Lecture 5 MWF 8
Yamani

Lecture 6 MWF 1
Taghavi- Chabert

INSTRUCTIONS

(a) Explain your answers precisely and clearly to ensure full credit.
(b) Calculators are not allowed.
(c) Use the backside of each page if needed.

Problem 1

(15 pts) Use the divergence theorem to find the outward flux of the field $\mathbf{F} = y\mathbf{i} + x\mathbf{j} + z^2\mathbf{k}$ through the surface of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$.

Problem 2

Given the ODE $y'' = -\frac{1}{4x}y' - \frac{1}{8x^2}y, \qquad x > 0$

(a) (5 pts) Show that $y_1 = \sqrt{x}$ is a solution of the given ODE.

(b) (10 pts) Find a solution y_2 that is linearly independent of y_1. (Verify that y_1 and y_2 are linearly independent). Conclude the general solution of the ODE.

Problem 3

(12 pts) Find an appropriate integrating factor to make the given ODE exact. Then solve the IVP

$$\begin{cases} (3x^2 + y + 2)\, y' = x \\ y(1) = 0 \end{cases}$$

Problem 4

(10 pts each) Solve each of the following IVPs.

a. $$\begin{cases} (x^2+1)\dfrac{dy}{dx} + xy = (x^3+x)y^3 \\ y(0) = 1 \end{cases}$$

(You may leave your solution in implicit form).

b. $\begin{cases} y' e^{-(x^2+\cos x)} = (2x - \sin x)(y^2 - 25) \\ y(0) = 0 \end{cases}$

(You may leave your solution in implicit form).

c. $$\begin{cases} \dfrac{dy}{dx} = \dfrac{\sqrt{1-(x+y)^2}}{x+y} - 1 \\ y(0) = \dfrac{1}{2} \end{cases}$$

(You may leave your solution in implicit form).

Problem 5

(15 pts) Use the substitution $y = e^{3z}$ to reduce the following ODE

$$6xe^{3z}\,dz - (x^2 e^{-3z} + e^{3z})\,dx = 0$$

to a homogeneous ODE. Solve the obtained homogeneous ODE. Then deduce $z(x)$, the explicit general solution of the given ODE.

Problem 6

a. (4 pts) Find the general solution of the ODE:

$$y''' + 2y'' + y' + 2y = 0$$

b. Do the following IVP's have a solution? If yes, is it unique? Justify your answer.

(i) (3 pts)

$$\begin{cases} y' = (x^2 + y^2)\sqrt{x-2} \\ y(3) = 3 \end{cases}$$

(ii) (3 pts)

$$\begin{cases} y'' = (x^2 + y)\sqrt{x-2} \\ y(3) = 3,\ y'(3) = 4 \end{cases}$$

(iii) (3 pts)

$$\begin{cases} y = (x^2 + (y')^2)\sqrt{x-2} \\ y(3) = --1 \end{cases}$$